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Gravitational Wave Detection by Bounded Cold Electronic Plasma in a Long Pipe

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Abstract We intend to propose an experimental sketch to detect gravitational waves (GW) directly, using an cold electronic plasma in a long pipe. By considering an cold electronic plasma in a long pipe, the Maxwell equations in $3+1$ formalism will be invoked to relate gravitational waves to the perturbations of plasma particles. It will be shown that the impact of GW on cold electronic plasma causes disturbances on the paths of the electrons. Those electrons that absorb energy from GW will pass through the potential barrier at the end of the pipe. Therefore, crossing of some electrons over the barrier will imply the existence of the GW.

Keywords Gravitational wave · Plasma · Detection · General relativity

1 Introduction

Stars and spaces between them are composed of matter in plasma phase. Since gravitational field is the most effective force in the macroscopic scale it will determine the dynamics of astrophysical objects. So it is necessary to study the effects of gravity on plasma phase of matter. For instance, the major reason for spiral shape of galactic arms has been known to be the effect of GW at the first stages of forming spiral galaxies [1]. Thus the theory of plasma state of matter in curved spacetime has been carried out [2–12]. An interesting phenomenon that happens is creation of electromagnetic field by GW. Gravitational fields enters into Maxwell equations as current terms, and like the electrical current create a electromagnetic field [12]. An interesting point in such studies is the emergence of resonance phenomenon in the interaction of GW with plasma [13]. This fact is used to propose a new method for detecting GW, beside standard methods [14, 15].

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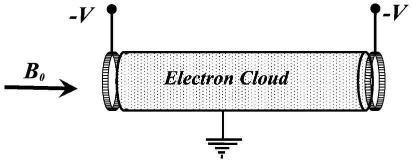

Fig. 1 Enclosed cold electronic plasma in a long pipe. Two negative potential rings at the end of the pipe create a potential barrier

As we know gravitational waves have not been detected directly in the laboratory. The very small value of gravitational constant in comparison with Coulomb constant causes such failures in detection of GW in laboratory efforts. There have been many efforts to suggest experiments in order to detect GW. Metal rod resonance and optical interferometry have been the most common methods in efforts to observe GW [14, 15]. There are some other methods too [16]. The most successful method implying the existence of GW has been the energy reduction of binary stars [17, 18]. In fact we have never observed the gravitational waves. Taylor has computed the energy of gravitational radiation of binary stars rotating about each other. Observed energy reduction of such stars is in full agreement with Taylor calculation.

The most promising idea is that of Weber. To observe GW effects directly in the laboratory, Weber has computed the effect of GW on the length of an Aluminum bar. We have used the basic Weber's idea but changed some ingredients. We thought that replacing Weber's rod with an electronic bar may be useful. Actually electronic bar has a great advantage; the electronic bar does not have the unwanted solid structure and then, atomic structures does not have noisy effects on it. Eliminating noisy effects is very important because they may interfere with GW. Our main purpose is to separate GW from environmental effects as much as possible. In order to do so, we enclose a cold electronic plasma in an potential well First, Fig. 1 and then two negative rings will be placed at the ends of the pipe to create a potential barrier. The potential barrier will enclose electronic cloud within the cylinder. It can be shown that the slope of this barrier is very sharp, so the trapped electrons will encounter to the potential barrier very close to the two rings [19]. The rings will create radial fields that will be discussed later. We will place a shield on the whole apparatus to prevent it from environmental noisy effect. Thus there is no any external electromagnetic field affecting on the pipe. But GW will pass through the metallic wall and will affect on the dynamics of the internal electrons. Length of the pipe should be suitably chosen [20–22]. Electrons within the pipe will get energy from GW and will eventually pass through the ring's potential barrier. Almost all electron are trapped in the pipe but small number of electrons can cross the barrier due to energy absorption from GW. The escaped electrons may be simply detected by an electronic detector.

In this paper we consider first bounded cold electronic plasma in a long pipe. The effect of Perturbation in electronic motion will be considered in Sect. 3. Then we review $3+1$ formalism to explain why GW has real existence. We will consider in Sect. 5 the effect of GW on cold electronic plasma that is enclosed in a long pipe. Finally we propose this apparatus for a possible detector of GW.

2 Enclosed Cold Electronic Plasma in a Long Pipe

We have previously studied the electric field inside a long pipe equipped two negative charge rings and shown that it is damped as cosine hyperbolic function [19]. Actually there can not be any sign of electric field created by the rings within the pipe. Suppose an amount of cold electronic cloud within the pipe is imported. We impose a longitudinal magnetic field B_0 to radially enclosed electronic cloud, Fig. 1. The electrons will rotate around the pipe axis. During rotation, they will move to the ends of the pipe. At the ends, the electrons will confront the barrier and will be reflected. We will use the fluid model to describe the situation, so there will not be pure longitudinal motion and any B_φ due to the longitudinal motion of electrons. Rotational motion of the electrons will create a B_z that will oppose the B_0 (Lens law). There is some E_r due to non-neutral nature of the electronic plasma. We will now compute this E_r field from Poisson equation:

$$\vec{\nabla}.\vec{E} = -4\pi e n^0 \quad \Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}(rE_r) = -4\pi e n^0 \quad \Rightarrow \quad E_r = -2\pi e n^0 r. \tag{1}$$

To obtain B_z, we use the Steady state Ampere law:

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}(-n^0 e\vec{v}). \tag{2}$$

Since the vector $\vec{v}$ merely have v_φ component, we conclude from above equation that

$$\frac{\partial B_z^0}{\partial r} = \frac{4\pi}{c} n^0 e v_\phi. \tag{3}$$

Thus longitudinal magnetic field will be changed with radius by the rotational electron motion and then, longitudinal magnetic field will never be a uniform field. We will begin with momentum equation to find rotational frequency of the electrons around axis. The plasma is so cold that there is no pressure or impact terms in momentum equation. Thus:

$$nm\left(\frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}\right)v = -en^0\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right), \tag{4}$$

where n^0 is the electrons density, m the electron mass, and e the electron charge. Electric field in the cylinder is

$$\vec{E} = E_r \overrightarrow{a}_r + E_z \overrightarrow{a}_z. \tag{5}$$

E_z is largely due to lateral potential rings and a little due to finiteness of electronic column. As we have quoted earlier [19], the E_z of the rings will rapidly be damped, thus E_z will only be non-vanishing closely near (in some millimeter) the ends. We will ignore the E_z that belongs to the finiteness of the electronic column. E_r can be obtained from (2). The $\vec{B}$ field has two components:

$$\vec{B} = B_\phi \overrightarrow{a}_\phi + B_z \overrightarrow{a}_z. \tag{6}$$

B_z is due to longitudinal motion that will be ignored in the fluid model, because the longitudinal current average is zero. B_z has two components too:

$$B_z = B^0 + B_z^0, \tag{7}$$

where B^0 is the confining longitudinal external field and B_z^0 is due to the rotational motion. B_z^0 can be ignored in comparison with B^0. We then have from (4):

$$\frac{1}{r}(v_\phi)^2(-\overrightarrow{a}_r) = \frac{-e}{m}\left(E_r + \frac{v_\phi B^0}{c}\right)\overrightarrow{a}_r. \tag{8}$$

And then:

$$\omega_r = \frac{v_\phi^0}{r} = \frac{\omega_c}{2}\left[1 \pm \left(1 - \frac{2\omega_p^2}{\omega_c^2}\right)^{\frac{1}{2}}\right], \tag{9}$$

where $\omega_c = \frac{eB_0}{mc}$ is the cyclotron angular frequency of pseudo-neutral plasma and $\omega_p = \sqrt{\frac{4\pi e^2}{m}}$ is the angular frequency of the electronic plasma. If the plasma was not pseudo-neutral then there would not be any E_r. So we can conclude from (4): $\omega_r = \omega_c$. It is obvious from (9) that if we did not ignore B_z^0 in comparison with B^0 then B_z would be changed with radius (3). If doing so, ω_c will vary with r too, and then ω_r vary with r. But the following calculation shows that the variation is so small:

$$\frac{\partial B^0}{\partial r} = 4\pi n^0 e v_\phi^0 = \frac{4\pi n^0 e}{c}\frac{\omega_c}{2}\left[1 \pm \left(1 - \frac{2\omega_p^2}{\omega_c^2}\right)^{\frac{1}{2}}\right] r = \frac{2\pi n^0 e^2 B^0}{mc^2}\left[1 \pm \left(1 - \frac{2\omega_p^2}{\omega_c^2}\right)^{\frac{1}{2}}\right] r \ll 1. \tag{10}$$

3 The Effect of Perturbation on the Dynamic of Electrons

Maxwell's Eqs. and Newton's Eqs. dominate on perturbations namely perturbation can not get arbitrary values. Suppose an external agitations cause a first order perturbation:

$$n = n_0 + n_1, \qquad \vec{B} = \vec{B}_0 + \vec{B}_1, \qquad \vec{v} = \vec{v}_0 + \vec{v}_1, \qquad \vec{E} = \vec{E}_1. \tag{11}$$

We ignored the constant part of E. Starting by the first order momentum equation, one finds:

$$\frac{\partial \vec{v}_1}{\partial t} + \vec{v}_0.\vec{\nabla}\vec{v}_1 + \vec{v}_1.\vec{\nabla}\vec{v}_0 = \frac{-e}{m}\left(\vec{E}_1 + \frac{\vec{v}_0 \times \vec{B}_1 + \vec{v}_1 \times \vec{B}_0}{c}\right). \tag{12}$$

Suppose the perturbation merely has radial and polar components:

$$\vec{v}^1 = v_\phi \overrightarrow{a}_\phi + v_r \overrightarrow{a}_r, \tag{13}$$

where it is a mode. There are many electromagnetic modes that correspond to excitant fields. We will see that in the first order, the shear tensor can only exist. Then only deformation in x and y components will exist. Expressing (2) in cylindrical coordinates:

$$\frac{\partial v_r}{\partial t} + \omega_r\left(\frac{\partial v_r}{\partial \phi} - v_\phi\right) - \omega_r v_\phi = \frac{-e}{m}E_r - \frac{e}{mc}(v_0 B_z + v_\phi B_0), \tag{14}$$

$$\frac{\partial v_\phi}{\partial t} + \omega_r\left(\frac{\partial v_\phi}{\partial \phi} + v_r\right) + v_r\frac{\partial v_0}{\partial r} = \frac{-e}{m}E_\phi + \omega_c v_r. \tag{15}$$

We can express the above equations in Fourier domain $\frac{\partial}{\partial t} = -i\omega$ and $\frac{\partial}{\partial \varphi} = ik$, and using $\frac{\partial v_0}{\partial r} = \omega_r$ approximation to obtain,

$$i(k\omega_r - \omega)v_r - \omega_r v_\phi = \frac{-e}{m}E_r - \frac{ev_0}{mc}B_z, \tag{16}$$

$$\omega_r v_r + i(k\omega_r - \omega)v_\phi = \frac{-e}{m}E_\phi. \tag{17}$$

If we solve this algebraic equations, we can express velocity in term of electric and magnetic field components

$$v_\phi = aE_r + brB_z + fE_\phi, \qquad v_r = AE_\phi + DE_r + SrB_z, \tag{18}$$

where a, b, f, A, D, S are defined as follows

$$\begin{aligned}
&a = \frac{-e\omega_r/m}{(k\omega_r-\omega)^2-\omega_r^2}, \qquad b = \frac{-e\omega_r^2/mc}{(k\omega_r-\omega)^2-\omega_r^2}, \qquad f = \frac{i(k\omega_r-\omega)/m}{(k\omega_r-\omega)^2-\omega_r^2}\\
&A = \frac{-e}{m\omega_r} + \frac{(k\omega_r-\omega)^2}{\omega_r m[(k\omega_r-\omega)^2-\omega_r^2]}, \qquad D = \frac{i(k\omega_r-\omega)e/m}{(k\omega_r-\omega)^2-\omega_r^2} = ef,\\
&S = \frac{ie\omega_r(k\omega_r-\omega)/mc}{(k\omega_r-\omega)^2-\omega_r^2}.
\end{aligned} \tag{19}$$

We need electron density and velocity to compute electromagnetic's field in the cylinder. Equation (18) shows that velocity can be expressed in terms of fields. Now we write density in terms of velocity. Since unperturbed velocity has only ϕ component, we have from linearized momentum equations;

$$\frac{\partial n_1}{\partial t} + n_0\vec{\nabla}.\vec{v}_1 + \omega_r\frac{\partial n_1}{\partial\phi} = 0. \tag{20}$$

Converting into Fourier domain gives:

$$-i\omega n_1 + n_0\vec{\nabla}.\vec{v}_1 + ik\omega_r n_1 = 0 \quad\Longrightarrow\quad n_1 = \frac{n_0\vec{\nabla}.\vec{v}_1}{i(\omega-k\omega_r)}. \tag{21}$$

Now, we begin with linearized Ampere equations and replace electron's velocity and density from (18) and (21). First order Ampere equations are

$$\vec{\nabla}\times\vec{B}_1 = \frac{1}{c}\frac{\partial\vec{E}^1}{\partial t} - \frac{4\pi e}{c}(n_0\vec{v}_1 + n_1\vec{v}_0). \tag{22}$$

Expressing (22) in cylindrical coordinates and then in Fourier domain:

$$\begin{aligned}
\frac{ik}{r}B_z - ik'B_\phi &= \frac{-i\omega}{c}E_r - \frac{4\pi en_0}{c}v_r,\\
\frac{-\partial B_z}{\partial r} &= \frac{-i\omega}{c}E_\phi - \frac{4\pi en_0}{c}v_\phi - \frac{4\pi ev_0}{c}n_1,\\
\frac{\partial}{\partial r}(rB_\phi) &= 0.
\end{aligned} \tag{23}$$

Now, we put n and v from (18) and (21) in (23):

$$\begin{aligned}
\frac{ik}{r}B_z - ik'B_\phi &= \frac{-i\omega}{c}E_r + (A'E_\phi + D'E_r + S'rB_z),\\
\frac{-\partial B_z}{\partial r} &= \frac{-i\omega}{c}E_\phi + (a'E_r + b'rB_z + f'E_\phi)\\
&\quad + \left[A''\frac{\partial}{\partial r}(rE_\phi) + D''\frac{\partial}{\partial r}(rE_r) + s''\frac{\partial}{\partial r}(r^2B_z)\right],
\end{aligned} \tag{24}$$

where

$$A' = \frac{-4\pi e n_0}{c} A, \qquad D' = \frac{-4\pi e n_0}{c} D, \qquad S' = \frac{-4\pi e n_0}{c} S,$$
$$a' = \frac{-4\pi e n_0}{c}\left(1 + \frac{k\omega_r}{\omega - k\omega_r}\right)a, \qquad A'' = \frac{-4\pi e\omega_r n_0}{ic(\omega - k\omega_r)} A. \tag{25}$$

We must eliminate one of E or B to solve (24). Using Faraday law in cylindrical coordinates, we can write B in term of E

$$B_\phi = \frac{ck'}{\omega} E_r, \qquad B_z = \frac{c}{ir\omega}\left(\frac{\partial}{\partial r}(rE_\phi) - ikE_r\right). \tag{26}$$

Putting (26) in (24) and solving E_ϕ, gives:

$$\begin{aligned}
&\left(\frac{-i\omega}{c} + f'\right)E_\phi + A''\frac{\partial}{\partial r}(rE_\phi) + a'M(r)\frac{\partial}{\partial r}(rE_\phi) - a'N(r)E_\phi \\
&= \frac{-c}{i\omega}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi)\right) + \frac{kc}{\omega}\frac{\partial}{\partial r}\left(\frac{M(r)}{r}\frac{\partial}{\partial r}(rE_\phi)\right) \\
&\quad - \frac{kc}{\omega}\frac{\partial}{\partial r}\left(\frac{N(r)}{r}E_\phi\right) - \frac{b'c}{i\omega}\frac{\partial}{\partial r}(rE_\phi) + \frac{kb'cM(r)}{\omega}\frac{\partial}{\partial r}(rE_\phi) \\
&\quad - \frac{kb'cN(r)}{\omega}E_\phi - \frac{-S''c}{i\omega}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(rE_\phi)\right),
\end{aligned} \tag{27}$$

where

$$M(r) = \frac{(\frac{ik}{r} - S'r)(\frac{c}{ir\omega})}{(\frac{-i\omega}{c} + D') + ik(\frac{ik}{r} - S'r)(\frac{c}{ir\omega}) + \frac{ik'^2 c}{\omega}},$$
$$N(r) = \frac{A'}{(\frac{-i\omega}{c} + D') + ik(\frac{ik}{r} - S'r)(\frac{c}{ir\omega}) + \frac{ik'^2 c}{\omega}}. \tag{28}$$

Solving (27) and putting the field equal to zero at $r = R$, we can obtain dispersion relation. Homogeneous second order differential (27) has two solutions:

$$E_\phi(r) = c_1 E_{1\phi} + c_2 E_{2\phi}. \tag{29}$$

One of this independent solution, for example $E_{2\phi}$, is singular at origin and then is not physical. Reminder field must satisfy the following boundary condition:

$$E_{1\phi}(r = a) = 0. \tag{30}$$

By solving the obtained algebraic equation, one should obtain desired dispersion relation:

$$k = k(\omega). \tag{31}$$

With series method we have solved (27) by MAPLE software. Based on what has been said before, we have obtained dispersion relation. The result is sketched in Fig. 2. One can clearly see the Langmuir and electromagnetic parts in Fig. 2. For better viewing we have changed

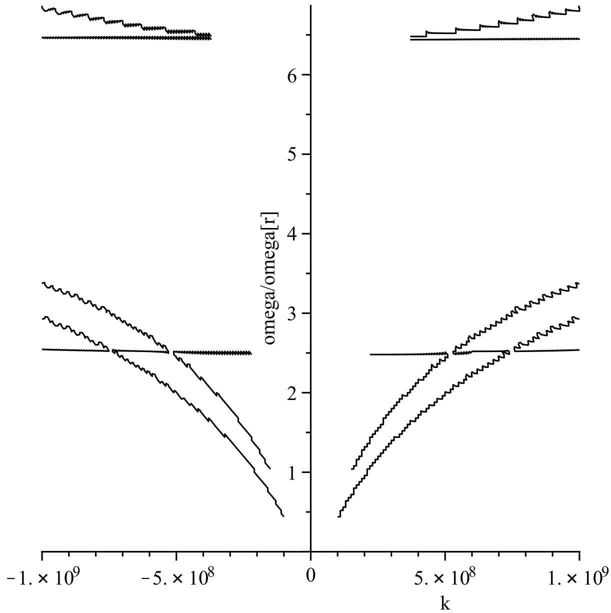

Fig. 2 Non relativistic dispersion relation, for; $n = 10^{20}\ \text{cm}^{-3}$, $l = 200$ cm, $B_0 = 10^8$ G

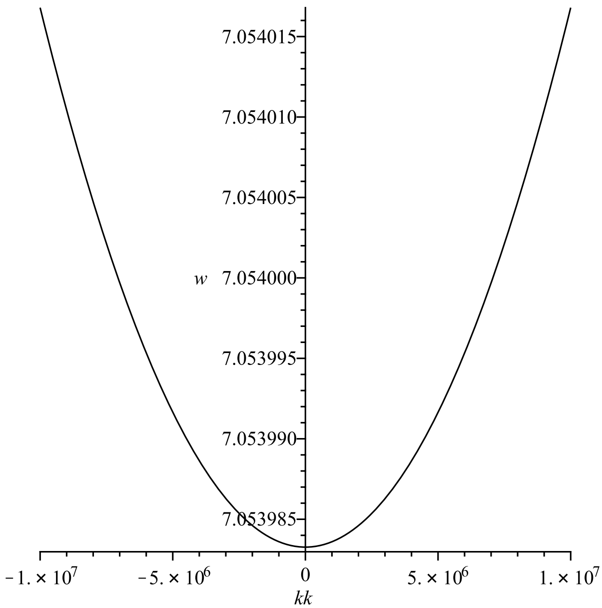

Fig. 3 Non relativistic dispersion relation. The electromagnetic part of Fig. 2

the limiting value in the MAPLE program to obtain Fig. 3. As we can see in Fig. 3, the cutoff frequency of electromagnetic wave is:

$$\omega_0 = 7.053\omega_r. \tag{32}$$

In this study we will suppose only transverse wave exist. This assumption is based on the fact that the shear tensor create transversal motion. Due to viscosity of the electronic

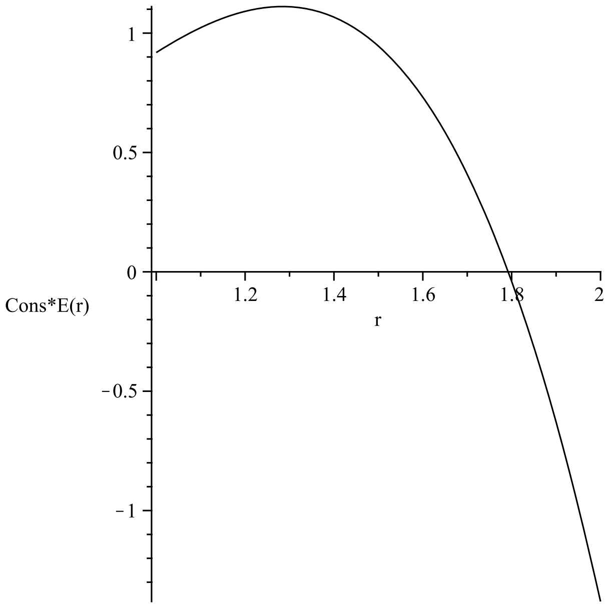

Fig. 4 E_ϕ variation with r, for; $n = 10^{20}$ cm^{-3}, $l = 200$ cm, $B_0 = 10^8$ G, and diameter of pipe is $r = 1.8$ cm

fluid, transversal motion will create longitudinal motion. We will also suppose that the electronic fluid is very dilute and hence there is not any longitudinal motion. Thus to first order, gravitational wave will create a consistent transversal mode. Note that we could use proper dielectric tensor:

$$\epsilon = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \omega_p^2/\omega^2 \end{pmatrix}. \tag{33}$$

In the magnetized non neutral electronic plasma confined in a metallic cylinder, the dielectric tensor is given by this matrix. With this dielectric tensor we arrive at dispersion relation [23]:

$$(ka)^2 = \frac{\omega^2 a^2}{c^2} - \frac{p_{n\nu}^2}{1 - \omega_p^2/\omega^2}, \tag{34}$$

where a is cylinder radius and $p_{n\nu}$ is νth root of first type Bessel function of order n. This dispersion relation has a cutoff frequency that is in full agreement with our previous result (32). Since we want to extend the problem to the relativistic case, we have not used dielectric tensor. In our consideration both Maxwell and momentum Equations have been changed. To compute electric field E_ϕ, we first obtain k from $k = \frac{2\pi n}{L}$, where L is the length of the pipe and n is an integer number. With a specified k, one obtains ω from dispersion relation and then electric field can be obtained. The result is sketched in Fig. 4.

We must note that E_ϕ is the solution of a homogeneous differential equation and hence it can be specified up to a multiplication factor.

4 3 + 1 Formalism

Although relativity joins space and time but sometimes splitting of space and time is useful. We usually like to think physical events as spacial events in different times. This is the 3 + 1 formalism; 3 for spatial dimensions and 1 for time dimension [24–30]. In $3+1$ formalism, every physical quantity splits to two components, one in the space like hypersurface and the other in the orthogonal direction to it. Energy momentum tensor in this formalism is [27, 31]:

$$T_{ab}^{dust} = \mu u_a u_b + q_a u_b + u_a q_b + p\gamma_{ab} + \pi_{ab}, \tag{35}$$

where u_a is the dust velocity component that have filled the whole spacetime manifold, μ energy density, q momentum density, p isotropic pressure, π traceless anisotropic pressure, and γ is projection operator. In this formalism, energy momentum tensor of Maxwell's field is [27, 32, 33]

$$T_{ab}^{(em)} = \frac{1}{2}(E^2 + H^2)U_a U_b + \frac{1}{6}(E^2 + H^2)\gamma_{ab} + 2Q_{(a}U_{b)} + P_{ab}, \tag{36}$$

where $E^2 = E_a E^a$, $H^2 = H_a H^a$, $Q_a = \eta_{abc} E^b H^C$ Poynting vector, and P_{ab} traceless tensor:

$$P_{ab} = P_{\langle ab\rangle} = \frac{1}{2}(E^2 + H^2)\gamma_{ab} - E_a E_b - H_a H_b.$$

E_a and H_a are the electromagnetic field components that can be obtained from Faraday tensor, F_{ab}, by

$$E_a = F_{ab}u^b, \qquad H_a = \frac{1}{2}\eta_{abc}F^{ab}. \tag{37}$$

Where η_{abc} is Levi-Civita tensor. In $3+1$ formalism, matter and energy will be specified by μ, q^a, P, π^{ab}, E^a, H^a, Q_a, and P_{ab}. Spacetime structure will be specified by, $\dot{u}_a$ fluid acceleration, θ expansion coefficient, E^{ab} electric Weyl tensor, and H^{ab} magnetic Weyl tensor. These quantities are defined as follows:

$$\begin{aligned} &\theta = \tilde{\nabla}_a U^a \quad (\theta = 3H), \qquad \sigma_{ab} = \tilde{\nabla}_{\langle a}U_{b\rangle}, \qquad \omega_{ab} = \tilde{\nabla}_{[a}U_{b]} \\ &E_{ab} = c_{acbd}U^c U^d, \qquad H_{ab} = \frac{1}{2}\eta_{ade}c_{bc}^{de}U^c, \end{aligned} \tag{38}$$

where $\tilde{\nabla}$ is projection of covariant derivative in hypersurface, c_{abcd} the Weyl conformal curvature tensor, and H the Hubble constant. If $\omega^{ab} = 0$, $\tilde{\nabla}$ will then equal to 3 dimensional covariant derivative in the hypersurface, in such case we denote it with ∇. Dynamic of energy-matter quantities and geometrical quantities can be obtained by projection of Einstein equation along normal vector (orthogonal to hypersurface). We list some of these equations that will be needed later [27]:

$$\begin{aligned} &\dot{\mu} + \tilde{\nabla}_a q^a = -\theta(\mu + p) - 2(\dot{U}_a q^a) - (\sigma_b^a \pi_a^b) \quad (\mu \text{ propagation Eq.}), \\ &\dot{q}^{\langle a\rangle} + \tilde{\nabla}^a p + \tilde{\nabla}_b \pi^{ab} = \frac{-4}{3}\theta q^a - \sigma_b^a q^b - (\mu + p)\dot{U}^a - \dot{U}_b \pi^{ab} - \eta^{abc}\omega_b q_c, \\ &\quad (q \text{ propagation Eq.}), \\ &\dot{E}_{\langle a\rangle} = \left(\sigma_{ab} + \eta_{abc}\omega^c - \frac{2}{3}\theta\gamma_{ab}\right)E^b + \eta_{abc}\dot{U}^b H^c + \operatorname{curl} H_a - j_a, \\ &\quad (E_a \text{ propagation Eq.}), \\ &\dot{H}_{\langle a\rangle} = \left(\sigma_{ab} + \eta_{abc}\omega^c - \frac{2}{3}\theta\gamma_{ab}\right)H^b + \eta_{abc}\dot{U}^b E^c - \operatorname{curl} E_a \quad (H_a \text{ propagation Eq.}), \end{aligned}$$

$$\dot{\sigma}^{\langle ab\rangle} - \tilde{\nabla}^{\langle a}\dot{U}^{b\rangle} = \frac{-2}{3}\theta\sigma^{ab} + \dot{U}^{\langle a}\dot{U}^{b\rangle} - \sigma_c^{\langle a}\sigma^{b\rangle c} - \omega^{\langle a}\omega^{b\rangle} - \left(E^{ab} - \frac{1}{2}\pi^{ab}\right)$$

$(\sigma_{ab}$ propagation Eq.),

$$\left(\dot{E}^{\langle ab\rangle} + \frac{1}{2}\dot{\pi}^{\langle ab\rangle}\right) - (\mathrm{curl}\, H)^{ab} + \frac{1}{2}\tilde{\nabla}^{\langle a}q^{b\rangle}$$

$$= \frac{-1}{2}(\mu + p)\sigma^{ab} - \theta\left(E^{ab} + \frac{1}{6}\pi^{ab}\right) + 3\sigma_c^{\langle a}\left(E^{b\rangle c} - \frac{1}{6}\pi^{b\rangle c}\right),$$

$$- \dot{U}^{\langle a}q^{b\rangle} + \eta^{cd\langle a}\left[2\dot{U}_c H_d^{b\rangle} + \omega_c\left(E_d^{b\rangle} + \frac{1}{2}\pi_d^{b\rangle}\right)\right] \quad (E_{ab} \text{ propagation Eq.}),$$

$$\dot{H}^{\langle ab\rangle} + (\mathrm{curl}\, E)^{ab} - \frac{1}{2}(\mathrm{curl}\, \pi)^{ab}$$

$$= -\theta H^{ab} + 3\sigma_c^{\langle a}H^{a\rangle c} + \frac{3}{2}\omega^{\langle a}q^{b\rangle} - \eta^{cd\langle a}\left[2\dot{U}E_d^{b\rangle} - \frac{1}{2}\sigma_c^{b\rangle}q_d - \omega_c H_d^{b\rangle}\right]$$

$(H_{ab}$ propagation Eq.), (39)

where bracket in tensor or vector means as:

$$v^{\langle a\rangle} = \gamma_b^a v^b \quad \text{and} \quad T^{\langle ab\rangle} = \left[\gamma_c^{(a}\gamma_d^{b)} - \frac{1}{3}\gamma^{ab}\gamma_{cd}\right]T^{cd}.$$

In deriving (39), we suppose that dust tensor prevail to electromagnetic tensor, also we have ignored gravitational self interaction. In $3+1$ formalism, E^{ab} and H^{ab} represent gravitational wave and σ^{ab} entered as an intermediate field [34–37]. In Minkowskian space $\omega = \theta = \dot{u} = \sigma = 0$ and (39) reduce to

$$\begin{aligned}
&\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{v}) = 0,\\
&\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{v}\right)^a = -\sigma_b^a - \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)^a,\\
&(\vec{\nabla}\times\vec{B})_a = \frac{-i\omega}{c}E_a + \frac{4\pi}{c}(-nq\vec{v})_a - \sigma_{ab}E^b,\\
&(\vec{\nabla}\times\vec{E})_a = \frac{i\omega}{c}B_a + \sigma_{ab}H^b.
\end{aligned} \tag{40}$$

Note that we have added Lorentz force to second equation of (39) to obtain the second equation of (40) [12]. As we can see from (40), the gravitational waves itself have not been entered in these equations, but correlated field σ have been appeared in these equations.

5 Gravitational Waves Effect on Bounded Cold Electronic Plasma

We have seen in Sect. 3 that, if for any reason the fields in the cylinder is perturbed then the resulting fields will satisfy certain homogeneous equations. Now, if we expose gravitational waves on the plasma, we will get a non-homogeneous deferential equations with gravitational wave term at the right hand side. We observe from dispersion relation that at a certain frequency, the motion of the electrons will resonate and can pass through the potential barrier. We follow Sect. 3 to obtain non-homogeneous equations. To do that we replace the leading equations of Sect. 3 with (40). To solve the resulting equations, we focus on an spacial case that σ^{ab} is diagonal. With this assumption, we write velocity field in term of electromagnetic field and gravitational field, σ:

$$v_\phi = aE_r + brB_z + fE_\phi + gr\sigma, \qquad v_r = AE_\phi + DE_r + SrB_z + Gr\sigma, \tag{41}$$

where

$$\begin{aligned}
a &= \frac{-e\omega_r/m}{(k\omega_r-\omega)^2-\omega_r^2}, \qquad b = \frac{-e\omega_r^2/mc}{(k\omega_r-\omega)^2-\omega_r^2} = \frac{\omega_r}{c}a,\\
f &= \frac{i(k\omega_r-\omega)/m}{(k\omega_r-\omega)^2-\omega_r^2}, \qquad g = \frac{i(k\omega_r-\omega)/e}{(k\omega_r-\omega)^2-\omega_r^2}\omega_r\\
A &= \frac{-e}{m\omega_r} + \frac{(k\omega_r-\omega)^2}{\omega_r m[(k\omega_r-\omega)^2-\omega_r^2]}, \qquad D = \frac{i(k\omega_r-\omega)e/m}{(k\omega_r-\omega)^2-\omega_r^2},\\
S &= \frac{ie\omega_r(k\omega_r-\omega)/mc}{(k\omega_r-\omega)^2-\omega_r^2}, \qquad G = \left[\frac{-e}{m\omega_r} + \frac{(k\omega_r-\omega)^2}{\omega_r m[(k\omega_r-\omega)^2-\omega_r^2]}\right]\frac{m\omega_r}{e} = A\frac{m\omega_r}{e}.
\end{aligned} \tag{42}$$

Now if we use (41) for the velocity field we can express magnetic field in term of electric field by the Ampere equations (40):

$$\begin{aligned}
\left(\frac{ik}{r} - S'\right)B_z - ik'B_\phi &= \left(\frac{-i\omega}{c} + D\right)E_r + A'E_\phi + G'r\sigma,\\
-\frac{\partial B_z}{\partial r} - b'rB_z - S''\frac{\partial}{\partial r}(r^2B_z) &= \left(\frac{-i\omega}{c} + f'\right)E_\phi + A''\frac{\partial}{\partial r}(rE_\phi) + a'E_r\\
&\quad + D''\frac{\partial}{\partial r}(rE_r) + g'r\sigma + G''\frac{\partial}{\partial r}(r^2\sigma),
\end{aligned} \tag{43}$$

where the primed and double primed quantity is defined as follow

$$A' = \frac{-4\pi en_0}{c}A, \qquad a' = \left(\frac{-4\pi en_0}{c}\right)\left(1 + \frac{k\omega_r}{(\omega-k\omega_r)}\right)a, \qquad A'' = \frac{-4\pi e\omega_r n_0}{ic(\omega-k\omega_r)}A. \tag{44}$$

If we put (43) in Faraday equations (40) and obtain E_φ from it, we will obtain the counterpart of (27):

$$\begin{aligned}
&\left(\frac{i\omega}{c} + f'\right)E_\phi + A''\frac{\partial}{\partial r}(rE_\phi) + a'M(r)\frac{\partial}{\partial r}(rE_\phi) - a'N(r)E_\phi\\
&= \frac{-c}{i\omega+\sigma c}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi)\right) + \frac{ikc}{i\omega+\sigma c}\frac{\partial}{\partial r}\left(\frac{M(r)}{r}\frac{\partial}{\partial r}(rE_\phi)\right)\\
&\quad - \frac{ikc}{i\omega+\sigma c}\frac{\partial}{\partial r}\left(\frac{N(r)}{r}E_\phi\right) - \frac{b'c}{i\omega+\sigma c}\frac{\partial}{\partial r}(rE_\phi)\\
&\quad + \frac{ikb'c}{i\omega+\sigma c}M(r)\frac{\partial}{\partial r}(rE_\phi) - \frac{ikb'c}{i\omega+\sigma c}N(r)E_\phi - \frac{S''c}{i\omega+\sigma c}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(rE_\phi)\right)
\end{aligned}$$

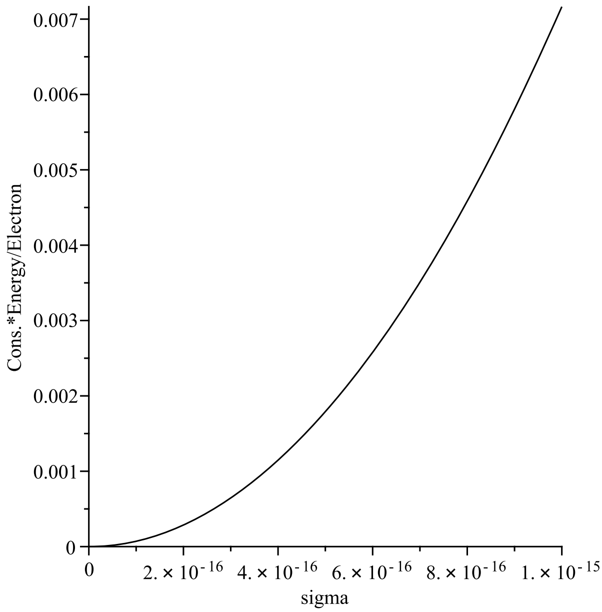

Fig. 5 Kinetic energy of electronic plasma particle will be increased by GW, for; $n = 10^{20}\ \text{cm}^{-3}$, $l = 200$ cm, $B_0 = 10^8$ G

$$+ (g' + 2G'')r\sigma - \frac{ikc\sigma}{i\omega + \sigma c}\frac{\partial}{\partial r}\left(\frac{L(r)}{r}\right) - \frac{ikb'c\sigma}{i\omega + \sigma c}L(r), \tag{45}$$

where $M(r)$, $N(r)$ and $L(r)$ are defined as follows:

$$M(r) = \left(\frac{ik}{r} - S'r\right)\left(\frac{c}{r(i\omega + \sigma c)}\right)\Big/ R(r), \qquad N(r) = A'/R(r), \qquad L(r) = G'r/R(r),$$
$$R(r) \equiv \left(\frac{-i\omega}{c} + D'\right) + \left(\frac{ik}{r} - S'r\right)\left(\frac{c}{r(i\omega + \sigma c)}\right) + \frac{ik'^2 c}{\omega}. \tag{46}$$

As what has been said in Sect. 3, $E_{\phi(r)}$ can be obtained from (45) and the $E_r(r)$ can be obtained from following equation:

$$E_r(r) = M(r)\frac{\partial}{\partial r}(rE_{\phi(r)}) - N(r)E_\phi(r), \tag{47}$$

and magnetic field is:

$$B_\phi = \frac{ck'}{\omega}E_r, \tag{48}$$

$$B_z = \frac{c}{r(i\omega + \sigma c)}\left(\frac{\partial}{\partial r}(rE_\phi) - ikE_r\right). \tag{49}$$

Velocity field are obtained from (41). Kinetic energy of an electron near the far end of the cylinder for $r = 4$ is sketched in Fig. 5. As we can see the kinetic energy will increase with external gravitational field.

According to general relativity the geometry of spacetime is determined by matter and energy. The motion of heavy mass in the spacetime causes time dependent variation in its curvature that is called GW. The effect of gravitation that entered in this study appears as σ field that are originated from the motion of far heavy (binary) stars. Having the order of magnitude of σ that belongs to the possible sources of GW, one can evaluate the kinetic energy of the electrons. If one knows the system parameters like electron density, the size of the pipe, and potential of the rings, he/she can in principle create such appropriate conditions that the electrons inside the pipe can pass through the potential barrier, that is a direct sign of existence of GW. This is main idea of the present paper. With a proper σ field strength, a typical electron can get enough energy to cross the potential barrier. This is a remarkable result because based on this calculation one can hope to detect the GW directly by the method described here.

6 Conclusions

We have proposed an experimental sketch to detect gravitational waves (GW) *directly*, using an cold electronic plasma in a long pipe. By considering an cold electronic plasma in a long pipe, the Maxwell equations in $3 + 1$ formalism have been invoked to relate gravitational waves to the perturbations of plasma particles. It has been shown that the impact of GW on cold electronic plasma causes disturbances on the paths of the electrons. Then we have shown that those electrons that absorb energy from GW will pass through the potential barrier at the end of the pipe. Therefore, crossing of some electrons over the barrier will imply the existence of the GW.

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References

1. Padmanabhan, T.: Theoretical Astrophysics, Vol. 3: Galaxies and Cosmology. Cambridge University Press, Cambridge (2002)
2. Elsasser, K., Popel, S.: Plasma Equation in General Relativity. American Institute of Physics, Melville (1997). S1070-664X 97 02007-7
3. Betschart, G.: Plasma physics on curved spacetimes. Thesis for the degree of Licetiate of Engineering, Department of Electromagnetics School of Electrical Engineering, Chalmers University of Technology, Göteborg, Sweden (2003)
4. Servin, M., Brodin, G., Marklund, M.: Cyclotron damping and Frady rotation of gravitational waves. Phys. Rev. D. **64**, 024013 (2001). gr-qc/0102031
5. Subramanian, K., Barrow, J.D.: Magnetohydrodynamics in the early universe and the damping of non-linear Alfven waves. Phys. Rev. D **58**, 083502 (1998). astro-ph/9712083 (1997)
6. Meier, D.L.: Ohms law in the fast lane: general relativistic charge dynamics. American Astronomical Society **605**, 340–349 (2004). astro-ph/0312053 (2003)
7. Zunkel, C., Betschat, G., Dunsby, P.K.S., Marklund, M.: On inhomogeneous magnetic seed field and gravitational waves within the MHD limit. Phys. Rev. D **73**, 103509 (2006). gr-qc/0602036
8. Moortgat, J., Kuijpers, J.: Indirect visibility of gravitational waves in magnetohydrodynamics plasma. gr-qc/0503074
9. Marklund, M., Dunsby, P.K.S., Betschart, G., Servin, M., Tsagas, C.G.: Charged multifluids in general relativity. arXiv: gr-qc/0211067 v2 19 Mar 2003
10. Brodin, G., Marklund, M., Dunsby, P.K.S.: Nonlinear gravitational wave interaction with plasma. Phys. Rev. D **62**, 104008 (2000)
11. Brodin, G., Marklund, M.: Parametric excitation of plasma waves by gravitational radiation. Phys. Rev. Lett. **82**, 3012–3015 (1999). astro-ph/9810128
12. Marklund, M., Clarkson, C.A.: The general relativistic MHD dynamo equation. Mon. Not. R. Astron. Soc. **358**(3), 892. astro-ph/0411140
13. Servin, M., Brodin, G.: Resonant interaction between gravitational waves. gr-qc/0302039

14. Weber, J.: General Relativity and Gravitational Waves. Interscience, New York (1961)
15. Gertsenshtein, M.E., Pustovoit, V.I.: Sov. Phys. JETP **16**, 433 (1962)
16. Chiao, R.Y.: New directions for gravity-wave physics via "Millikan oil drops" (2007). gr-qc/0610146v16
17. Hulse, R.A., Taylor, J.H.: Discovery of a pulsar in a binary system. Astrophys. J. **195**, L51–L53 (1975)
18. Weisberg, J.M., Taylor, J.H.: Relativistic binary pulsar B1913 + 16: thirty years of observations and analysis. ASP Conf. Ser. **328**, 25 (2005)
19. Rouhani, S.: Measurement of gravitational acceleration of antimatter. International Center for Theoretical Physics, Report No. Tc/89/405
20. Chiao, R.Y.: In: Barrow, J.D., Davies, P.C.W., Harper, C.L. Jr. (eds.) Science and Ultimate Reality, p. 254. Cambridge University Press, Cambridge (2004). quant-ph/0303100
21. Chiao, R.Y., Fitelson, W.J., Speliotopoulos, A.D.: Search for quantum transducers between electromagnetic and gravitational radiation: a measurement of an upper limit on the transducer conversion efficiency of yttrium barium copper oxide. gr-qc/0304026
22. Chiao, R.Y., Fitelson, W.J.: Time and matter in the interaction between gravity and quantum fluids: are there macroscopic quantum transducers between gravitational and electromagnetic waves? In: Bigi, I., Faessler, M. (eds.) The Proceedings of the 'Time & Matter Conference' in Venice, Italy, 11–17 August 2002, p. 85. World Scientific, Singapore (2006). gr-qc/0303089
23. Krall, N.A., Trivelpiece, A.W.: Principle of Plasma Physics. McGraw-Hill, New York (1973)
24. Thorne, K.S., Macdonald, D.: Black-hole electrodynamics—an absolute-space/universal-time formulation. Mon. Not. R. Astron. Soc. **198**, 339 (1982)
25. Gourgoulhom, E., Jaramillo, J.L.: A perspective on null hypersurfaces and isolated horizons. Phys. Rep. **423**(4–5), (2005). gr-qc/0503113 v1
26. Ellis, G.F.R.: General Relativity and Cosmology. Academic Press, New York (1971)
27. Ellis, G.F.R.: Cargese Lecture in Physics. Gordon & Breach, New York (1973)
28. Smarr, L.L., York Jr., J.W.: Kinematical condition in the construction of spacetime. Phys. Rev. D **17**, 2529–2551 (1978)
29. Poisson, E.: A Relativists Toolkit. Cambridge University Press, Cambridge (2004)
30. Choptuik, M.W.: The 3 + 1 Einstein equations. Unpublished Lecture Note: Relativity Theory II, Lectures 6, 7. Feb. 5, 10 (Spring 1998)
31. Landau, L.D., Lifshit, E.M.: Fluid Mechanics, 2nd edn. Pergamon, Elmsford (1987)
32. Tsagas, C.G.: Electromagnetic fields in curved spacetimes. Class. Quantum Gravity **22**, 393–407 (2005). gr-qc/0407080
33. Thorne, K.S., MacDonald, D.: Electrodynamics and curved spacetime: 3 + 1 formulation. R. Astron. Soc. Mont. Not. **198**, 339 (1982)
34. Maartens, R., Ellis, G.F.R., Siklos, S.T.C.: Class. Quantum Gravity **14**, 1927–1936 (1997). gr-qc/9611003
35. Dunsby, K.S., Bassett, B.A.C.C., Ellis, G.F.R.: Class. Quantum Gravity **14**, 1215–1222 (1997). gr-qc/9811092
36. Challinor, A.: Microwave background anisotropies from gravitational waves: the 1 + 3 covariant approach. Class. Quantum Gravity **17**, 871–889 (2000). astro-ph/9906474
37. Ellis, G.F.R., Hogan, P.A.: The electromagnetic analogue of some gravitational perturbation in cosmology, University of Capetown preprint (1996)